THE MATHEMATICS TEACHER

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THE MATHEMATICAL PESSIMIST.

BY W. R. RANSOM.

By the mathematical pessimist I mean the discouraged student, the boy (or girl) who has arrived at the conclusion that mathematicians, like poets, are born and not made, and that none was born on his birthday. He believes himself to be what Klein has called an "amathematician." We may quote the very words of the pessimist: you recognize the species when he refers to his geometry or his algebra—oh especially his algebra!—as "this stuff." Such a one was the writer of a charming Christmas story who began it with the statement that she could not bear the abbreviation "Xmas" because, she said, "X always seemed to me to stand for what was hard and difficult, unknown and unknowable."

I do not mean the happy blunderer who cannot get his answers right, the wild Indian whose tomahawk removes the radical from $\sqrt{a^2+x^2}$ with a swoop that takes the exponents off with it;—nor the young sinner who on meeting the equation $\sin x = 0$ gets rid of his sin by dividing it out, and for penance must lose the π to which he was entitled;—nor the girl who remonstrated with me when I agreed that $abc2x = 2abcx$ but held that $\sin 2x$ does not equal $2 \sin x$. "Why," said she, "I thought that when you learned a thing in mathematics it was always so."

No these persons are not your true pessimists. They are in their way good mathematicians, but not very profound and some-

what blundering. They forget the rules and reason by analogies which no doubt are more convincing to them at the moment than genuinely rigorous "epsilon-arguments" are to many a graduate student. Such persons are much more numerous than the pessimists and equally deserving of having papers written about them. I did not write about them for this reason: I know all about the bunglers and I should have so authoritatively settled their problems in a few pages that the subject would be finally closed. About the treatment and cure of pessimism, however, I know so little that I must infallibly say things so wrong as to provoke discussion. In this way I have hoped that this paper might lead to a profitable discussion from which I might learn many things that I am very anxious to know.

The pessimist is strong in his conviction. May he not sometimes be right? Maybe some people have a limit of mathematical power, as certainly all have of physical power. The limit may be at the first chapter of the algebra, or the line may be drawn still lower. There may be no such line: it may be a wholly imaginary line and yet to the poor pessimist be as terrible and grievous as the Equator to the little girl who defined it as a "menagery lion running around the earth."

We have all seen so often the wonderful change of mind that comes in school with a change of heart, the transformation of the hopeless student into the successful one, that most of us are reluctant to believe that any student is hopeless. In fact we are Optimistic about the worst of the Pessimists.

Professor Felix Klein says that the question whether there *are* "amathematicians" must be left to the psychologists to determine. Professor Julian Coolidge, answering an advocate of "practical mathematics only," said that we all study mathematics in the schools "because our minds are incurably mathematical." At the Albany meeting last November, Commissioner C. F. Wheelock gave statistics proving that low averages in mathematics are due to poor teaching rather than to the difficulties of the subject and he seemed to conclude that there are no cases of "amathematicism." Professor M. J. Babb agreed with this conclusion and gave excellent advice on improving both the teaching and the selection of topics, and on training the students to unite "chalk and talk." So far the authorities disagree with the pessimist's belief in his "amathematicism."

I am inclined to think the question still an open one. It may indeed turn out that none are by nature devoid of geometrical and analytical powers. Yet I should still fear that some pupils may grow into habits of thought which make the further cultivation of mathematics during their few schoolable years a practical impossibility.

We do not need to settle the question. Experience shows that some pessimists are curable. What then is the cure?

The personality of the teacher is the most potent aid. If a pupil believes that the teacher is thoroughly interested in his subject and thoroughly interested in the pupil, a link of the strongest kind has been made between the subject and the pupil. "Love me, love my dog" is good doctrine, even if the dog's name is Algebra. There is not much I can *say* about this: it is something to *be*. And I suppose we are all trying to be it as hard as we can, to see with our pupils' eyes, to feel with their hearts, that they may see with our eyes, feel with our hearts, and think with minds not perversely different from our own.

Professor Frank E. Seavey, of the English department at Tufts College, a man of rare ability in winning the confidence of college boys, tells me that we mathematics teachers, and even such of our students as get *A*'s and *B*'s cannot understand how things look to the fellow near the firing line, the student who works hard and gets a bare pass or fails.

The mathematical way of looking at things is abhorrent to the good sense of some otherwise minded folks. The proof for the non-existence of a maximum integer which begins "consider all possible things" provokes laughter. Such dicta as that zero divided by zero is not clearly equal to one, that there is no tangent of $90°$, that a curve without width can fill up an area, that the numbers $+1$ and -1 are just as deserving of the epithet *imaginary* (in its non-technical sense) as the number $\sqrt{-1}$, that a line segment may have a last point but no next to the last point, that there are as many points in an inch of line as in a mile,—these and many other paradoxes are rich and satisfying food to your pure mathematician, but caviar to the general. And to the pessimist? The complacency with which we swallow such "absurdities" is the clearest proof that higher mathematics is a delusion.

I have had long and earnest arguments with mature and gifted gentlemen, who had stood by no means low in their college mathematics,—with one because he would not admit the modern geometer's right to use the phrase "point at infinity" when clearely there is no such place as infinity, and moreover the thing referred to is not a point at all; with another because he would not allow the logician to translate the words "some lawyers are honest" by symbols that would exclude what all the world knows is implied, that *some are not!**

Some men of great force fail to see anything in important branches of intellectual theory. Some condemn philosophy, some theology, some political economy as delusions. Sir William Hamilton's opinion that the study of mathematics lacks most of the qualifications we should claim for it is warmly endorsed by a department head in the Massachusetts Institute of Technology. Commissioner David Snedden would greatly reduce the teaching of mathematics in our high schools. If mature and thoughtful men may so quarrel with us over fundamentals, need we doubt that Professor Seavey is right in denying us the entrée into the minds of some of our students?

Perhaps it is well for us to be more humble in our attitude toward those who scorn our science. Professor DuBois, of Atlanta, fresh from the Congress of Races at London in 1911, told us that the most significant note at that great meeting was this appeal to the white men from the "lesser breeds without the law": "When you come to our lands to teach us the wonderful things that science has done for your civilization, do not forget that we too have a civilization or our own, developed to fit our needs, that some of our ways are better for us than your ways, that some of our thoughts are more worthy than yours. Do not insist on teaching us only: learn of us too. And let us meet not as lord and vassal, but as friends, ready to take as well as give, each receiving the other's best, and yielding honorably to each other."

Is not the teacher always in danger of playing white man to the little heathen in his classes? Do you know the school where the motto of the mathematics department is that line from Kip-

* The Mate got even with his Captain by entering once in the log book: "Captain Evans was sober all day to-day!"

ling's Mandalay, "Lord, what *do* they understand?" How much better the attitude of Chaucer's Clerk: "and gladly wolde he lerne and gladly teche."

Enumerating the sorts of men whose thoughts go wrong, Locke says: "The third sort is those who readily and sincerely follow reason, but for want of having that which one may call large, sound, roundabout sense, have not a full view of all that relates to the question. . . . They have a pretty traffic with known correspondents in some little creek . . . but will not venture out into the great ocean of knowledge."

This description is applicable too often to the teacher of algebra, who usually knows the algebraic routine perfectly but has no unbookish nor even other-bookish applications connected with it, no interests in the broad world of science through which runs his narrow algebraic creek.

It is only by continuing to be a student that one can sympathize with students. The teacher should spend less time in correcting papers and more in study. Master the elements of many new subjects. "Thou that teachest another, teachest thou not thyself?" Every winter attack some substantial study. A good book (not too long) to dig it out of, and a fellow teacher to vie with one and keep one hard at it are quite necessary.

It may be that some teachers do not realize what a field there is for this sort of work, or what fun and profit it yields. I will suggest some books, not beyond the abilities and well connected with the interests of the average high school teacher of mathematics, which, besides offering such obstacles to a too easy reading as are importatn for the purpose, will richly reward the labor spent upon them.*

Worthen: Dynamics of Rotation, Longmans.
Whitehead: Introduction to Mathematics, Holt.
Evans: Teaching of Mathematics, Houghton Mifflin.
Young: Fundamental Concepts of Algebra and Geometry, Macmillan.
Bonola: Non Euclidean Geometry, Open Court.
Manning: Irrational Numbers, Wiley.
Fine: Number System of Algebra, Leach Shewell and Sanborn.
Coffin: Vector Analysis, Wiley.

* See also *The Teachers College Bulletin* No. 11, Second Series, Jan. 28, 1911: Bibliography of books on mathematics suitable for a high school library.

Risteen: Molecules and the Molecular Hypothesis, Ginn.
Airy: Undulatory Theory of Light, Macmillan.
Poincaré: Science and Hypothesis, Scribner.
Klein: Famous Problems in Elementary Geometry, Ginn.
Hanus: Determinants, Ginn.
Johnson: Curve Tracing, Wiley.
Young: Monographs on Modern Mathematics, Longmans.
Manning: Non Euclidean Geometry, Ginn.
Richardson and Ramsay: Modern Plane Geometry, Macmillan.

Moreover, teachers who have had French or German but do not think they have a reading knowledge of either will do well to undertake one of the excellent small books in those languages. These should be read aloud, not translated except as a last resort in occasional paragraphs. Many of our teachers believe they are confined to English as earnestly as the pessimist believes he is shut out of higher mathematics. Such will profit greatly by the struggle and will learn with surprise how easy it is to convert a poor literary knowledge of the foreign tongue into a fair reading knowledge of mathematical French or German. Begin with a book on a very elementary subject like Plane Geometry, or beginners, algebra, where there are plenty of diagrams or equations to help out and where the mathematical vocabulary is developed as you go along. For instance:

Mahler: Ebene Geometrie, Nr. 41, Sammlung Göschen.
Schubert: Arithmetik und Algebra, Nr. 48, Sammlung Göschen.
Lemoine: Geometrographie, No. 18 in Scientia, Gauthier-Villars.

As the second venture in French try

Laurent: La Geometrie Analytique General, A. Hermann.
Couturat: L'Algebre de la Logique, No. 24 in Scientia.
Barbarin: Geometrie Non Euclidienne, No. 15 in Scientia.

Further suggestions in German are

Böger: Elemente der Geometrie der Lage, G. J. Göschen.
Doehlemann: Projektive Geometrie, Nr. 72, Sammlung Göschen.
Beutel: Algebraische Kurven, Nr. 435–6, Sammlung Göschen.
Valentiner: Vektoranalysis, Nr. 354, Sammlung Göschen.
Jaeger: Theoretische Physik, Nr. 76–7–8, and 374, Sam. Göschen.

The Sammlung Göschen is a remarkable collection of small books, written in a masterly manner, costing only twenty-five cents apiece, yet fully equal to English books costing six times as much. B. G. Teubner is getting out a similar series. The

Scientia series of Gauthier-Villars, costing forty cents each, includes other interesting books not noted above.*

It seems to me a misfortune that our own countrymen do not produce more *small* books on advanced mathematical subjects, distinctly intended for those who are not specialists. The lack of such books lends color to the prevailing belief that mathematics as a sport is like football and not to be indulged in after graduation except by professionals.

Subjects of live interest and great fascination like projective geometry, non-Euclidean geometry, logical algebra, elementary function theory, infinite assemblages, geometrical transformations, deserve to be treated not too profoundly, in small volumes intended primarily for persons with the average ability of the mathematics teacher in the small high school. Our first rate mathematicians are all more eager to explore remote corners of the field of knowledge. The building of easier roads through already explored country, with occasional far-seeing glimpses into new country, such as French writers especially give us, seems to me a task worthy the mettle of our best men. I regret that the *Annals of Mathematics* has abandoned its plan of giving occasional papers in which old discoveries were made more accessible rather than new theories disclosed.

Such a book ought not to be as difficult reading as (say) Townsend's translation of Hilbert's "Fundamentals of Geometry," or some of the Monographs in Young's collection; but it should be closely reasoned, rich in information, and not so abstract in point of view as to repel the class of readers of whom I have been speaking. The knowledge that people who do not have to are reading mathematical books is a check upon the crop of pessimists. The teacher, honestly laboring over such books, frequently baffled but usually triumphant, may hope to keep in truer sympathy with his less proficient pupils.

* Professor J. T. Rorer adds these from an Italian series costing about thirty-five cents each:

Pincherle: Algebra Complementare, Parts I. and II. and Esercizi.
Pincherle: Geometria pura elementare.
Pincherle: Geometria pura elementare (Esercizi sulla).
Pincherle: Geometria metrica e trigonometria.
Alasia: Geometria e trigonometria della sfera.
Rossotti: Formularis Scolastico di Elementar Matematica.
Pascal: Determinanti e applicazioni.

By the introduction of more *practical* problems into mathematics courses much is being done to stimulate the interest and reduce the pessimism of a large class of students. Like all good things the use of practical problems is apt to be overdone when first taken up. Some of the problems which are most practical at the glazier's, the tin-smith's, or the naval architect's, prove to be the most impractical in the mathematics classroom. A practical problem is one which does not have to be lugged in and explained but one which slides up and irresistibly confronts you demanding a solution. I am a believer in practical problems, but my definition of them has changed since I first began collecting them. I now think that proving the rule for casting out nines, or finding what is the conclusion that can properly be drawn from the premises

No cat has two tails
One cat has one more tail than no cat

and interesting puzzles, paradoxes, and fallacies are quite as practical in the classroom as steam-fitter's difficulties.

I suppose that most of our pessimists become such during the algebra course. If our algebra is too abstract (and I believe it is) I think the remedy lies outside the algebra classroom as well as within. Let me speak of the value of casual algebra.

Other teachers should not wait for the algebra class to work up a thorough lay-out of algebraic usage, but should make use at will of simple algebraic equations and formulas as early as desired. I was taught cube root by means of the formula $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ before I ever heard of rules for algebraic multiplication or the binomial theorem. I suppose that modern algebra is what it is today, a universal language, recognizably the same in Russia and China as in New York, because it embodies the most natural* and inevitable way of representing numerical relations. Let a teacher of arithmetic, or carpentry, or what not, be encouraged to put an explanation into algebraic form whenever it is convenient to do so, and help the student to a more natural entry into algebra. If algebra produces most of the pessimists it is because it makes prematurely too ample preparations for solving complicated and difficult problems.

* Contrast the uniform interpretation of coefficients, exponents, and parentheses with the variety of notations and interpretations for derivatives in calculus, angles in geometry, and products in vectors.

Dr. E. V. Huntington says in his encyclopedia article on Symbolic Logic: "The only difficulty is to find any problems that are hard enough for this branch of the subject to attack." Every-day out-of-school experience says the same thing of all the harder parts of algebra. Dr. W. F. Osgood confesses that "the applications of algebra are to — algebra!" The problems of arithmetic and physics are for the most part absurdly simple. Algebraic analysis brings out this simplicity, and predisposes the student to study a mechanism that works so smoothly.

The casual teaching of algebra has worked well enough as I have seen it. We always have in our engineering school a good many students who are not very skillful algebraically. We used to put them into a course in that subject their first half year. Five years ago we discontinued this course. Now we get along as well as ever. We teach whatever algebra occasion demands at any time. We have got rid of the deadening effect and depressing influence of that course, and we cover more ground in two years than we could formerly, and have greatly improved the attitude of our students toward their prescribed mathematics.

In regard to changes affecting subject matter, I wish to add to the excellent proposals of Professor Babb. After reducing the emphasis on factoring, radicals, complex fractions, L.C.M., and G.C.D. in the beginners' course we can do something to bring into higher relief certain other topics.

One that I wish to call attention to is variation. In most algebras it gets very scant treatment. I think it is a subject of great fruitfulness, wholly typical of the algebraic method, and rich in applications of the every-day sort, and continually reappearing in geometry and higher mathematics.

Another is inequalities. The contrast between inequalities and equations helps break up the formalism which so easily degenerates into pessimism, and affords the student interesting opportunities for testing his own power of reasoning as to what operations are permissible. Teachers of physics treat by equations many problems in statics and in friction where a treatment by inequalities would be more adequate, more honest, and clearer to the student.

A third subject on which I would lay great stress has not even a name in our algebras. I will call it *reversal equations.* By

this term I mean *equations which contain the unknown only once*, so that to solve we have only to consider what operations have been performed upon the unknown and then to reverse each in reverse order. This class of equations has not only been unnamed but so far as I know has been nowhere recognized as an important class: yet it includes almost all the equations met with in elementary applied mathematics.* It is a class in which the analysis of the problem is peculiarly simple and natural, and in which the necessity for independent reasoning and the utility of the algebraic formulation are united in a combination which is ideal for pedagogic purposes. I consider that in calling attention to the claim of *reversal equations* to recognition as a *class* of equal importance with linears and quadratics I have made my chief contribution to algebraic progress.

Finally, if we secure the desired personality in the teacher, and if we get him into difficulties studying higher mathematics so that he can better appreciate the difficulties of his Freshmen, and if we increase the casual teaching of algebra, and redistribute the emphasis upon topics, and still have left a residue of pessimists—shall we let them go?

No doubt we waste a great deal of time trying to teach the unteachable (as philosophers do in trying to unscrew the inscrutable)—shall we merely solace ourselves with the comfort that to furnish thoughts for non-thinkers is a great achievement and worth the cost of much wasted by-product?

We cannot at present tell in advance the hopeless cases. I wish you might all have heard with me in Boston† Professor David Eugene Smith's remarkable parable in which the unpromising boy is all but prevented from ripening into a Thales, a Pythagoras, a Newton.

But if the psychologists do succeed in sifting out the "amathematicians" from among the pessimists, I hope we shall not have to remember and reproach ourselves for having forced too many complex fractions and radicals into their undigesting systems.

TUFTS COLLEGE.

* $s=\frac{1}{2}gt^2$, $A=p(1+r)^n$, $V^2=2gs$, $\sin\theta=\frac{1}{2}$, $x^{\frac{1}{2}}=3$, $V=\pi D^3/6$, etc.

† See "Some problems in the Teaching of Elementary Mathematics" in the MATHEMATICS TEACHER for March, 1913, page 161.

USE OF TEXT BOOKS IN THE EXAMINATION IN TRIGONOMETRY.

By B. C. Van Ingen.

In presenting this subject, I have attempted to get the opinion of representatives of three classes of people vitally interested in this change in the form of examination in trigonometry, viz., the instructor of trigonometry in the high school, the professors in the departments of mathematics in the colleges who build upon the preparation made by the high school pupil in this subject, and third the engineers who have had these courses in high school and college and have supplemented them by years of practical experience in putting into use these fundamental principles both personally and with assistants of less experience and of fresh book knowledge.

I sounded the opinion of the high schools by sending to several strong schools in different parts of the state the following letter:

"On the Regents examination paper in trigonometry last June was printed the following: 'Students taking this examination may use text books and notes prepared previous to the examination, but they must not communicate or use material prepared by another after the examination has begun.'

"1. What effect is this ruling having on the scholarship of your classes in trigonometry?

"2. Do you consider this a wise ruling? Why?

"3. What changes would you suggest in the *amount* of help allowed?

"4. What change would you suggest in the *kind* of helps allowed?

"5. Have you any changes you would like to see in the note at the head of this examination paper?"

Most of the answers to the first question were to the effect that there was no appreciable change in scholarship up to the present time.

In answer to the second question most replies were that the ruling was wise for this method is more of a test of the appli-

cation of power on the part of the pupil not a mere memory test, that it is practical and the method of life. Some suggested that the helps would tend to slow down the pupil on examination by wasting time in hunting for cases in the book similar to the ones on the examination paper. Those who suggested changes in the *amount* of help allowed would eliminate the text book but allow other prepared data which were enumerated under the answer to question four as notes, data, more difficult formulae, etc.

In answer to the question five suggestions were given that the note at the head of the Regents paper should read to allow use of notes, outlines, data, etc., prepared by the student before the examination—not allow the use of text book.

So much in regard to opinion of high school men.

The opinion of colleges seem to be unqualifiedly against the use of text book helps. One says: "Not at all in favor of recent ruling;" another, "Makes poor students in advanced Mathematics."

I quote in full the reply from the Chairman of Department of Mathematics in Cornell University.

"In reply to your recent inquiry concerning the wisdom of conducting examinations in trigonometry with the use of the text book and of such other helps that the candidates may have prepared, I would say that the mathematical staff of Cornell University is *not* in favor of the scheme, *unless* a separate test of the theoretical part of the subject can be provided. Under the proposed plan, we are afraid that too much emphasis will be put on numerical work, and upon the more difficult formulas; both of these may be excellent for certain purposes, but they are *not* the most valuable as a preparation for *further* study of mathematics. *Dexterity* and *accuracy in numerical* computation is now more generally taught than *skill in easy manipulation*; it is very unusual for a student who takes our entrance examination to show himself deficient in the *numerical* work. The more *difficult formulas* are only of *infrequent* occurrence, hardly justifying much time or effort being put upon them. What we desire is a ready knowledge of the elementary relations, such as complements, addition formulas, half angles, etc., and we fear that the new plan will not accomplish the purpose of securing this end."

Another college professor of mathematics says:

"I am not at all in favor of the present ruling in this matter. I would not allow them any help outside of a good table of logarithms. We find that boys who are taught to prepare for such examinations make poor students in advanced mathematics."

I wish to quote a few selections from letters from engineers. These all have wide experience with engineering problems of the office and of the field, and have numbers of assistants under them.

"I consider the free use of text books in trigonometry examination by high school students highly injudicious and dangerous. Preparation for "engineering" courses would, in my opinion, be weakened thereby, assuming, of course, that the work would not be covered again subsequently with a different kind of a test. . . . Trigonometry is one of the branches of mathematics of which the engineer must have a *firm* grasp and an *understanding* grasp rather than a rote knowledge. . . . Anything that permits or may possibly allow "getting by" without requiring a thorough knowledge of trigonometry is detrimental. I cannot conceive of an examination in trigonometry permitting the use of books which would correctly test the knowledge of the subject. One should know much more about the subject than where to find it in a book." Several refer to the practice in recent years adopted by the civil service in giving a part of the examination without helps of any kind and another part using helps freely, thus showing whether or not an engineer knows how to use a part of his tools, viz., his books, etc.

Another engineer says: "If it is desired to ascertain the student's knowledge of certain fundamental formulæ which he should know thoroughly, books should not be allowed. This might form the first part of an examination," Then he suggests another part of the examination in which great freedom be allowed in use of all kinds of aids.

The following letter is from a city engineer of wide experience: "In the matter of use of self prepared data and text books, I should say that for examinations in *elementary* trigonometry no text book or data be permitted but rather that the student be obliged to make use of his knowledge of the functions of an angle, as readily as he would the multiplication table

of the olden days. But when the less simple solutions are considered, it takes a very little time in practical life to either forget the more involved formulæ or, what is easier still, to forget, the origin or evolution of them. Aside from the undesirability of being obliged to remember long formulæ, the probability of a wrong statement of them is so great, that, speaking from my own experience, I should unhesitatingly say, use text books, by all means. After all is said and done, it seems to me that a technical education (aside from certain elementary things never to be forgotten) is but a matter of where most readily to put your hands on the best information, in the best form, for any given piece of work under consideration."

From these then it seems that high school teachers are divided in opinion as to use of text books; the colleges are opposed to the plan and practical engineers are opposed to the *free* use of the book but advocate its restricted use.

From experience and from the opinions expressed in letters and orally of others I would like to see the text book as an aid either eliminated, or better still restricted to use in a part of the examination only.

If not used at all, then pupils should, of course, have the tables, and the formulæ that are long and more or less involved and difficult to remember, as one engineer says possibly about 15 in number. They should not have access to solved numerical problems.

It seems that an examination could easily be arranged consisting of two parts—one using no aids, based upon the fundamentals, work to be handed in at end of a certain specified time or if finished at any time before. The other part to be done with any available help as text, data, etc.

I do not think this is catering to the thought so frequently expressed that we "pamper" the students and make easy the work and graduate pupils on the lowest terms possible.

I think it would be emphasizing the importance of the fundamentals, just as we have the old fundamental arithmetical tables, and at the same time be giving freedom to show skill in the use of the tools used in the practical applications of these fundamentals.

High School,
Frankfort, N. Y.

HOW SHOULD SECONDARY MATHEMATICS FOR GIRLS DIFFER FROM THAT FOR BOYS?

By Jean F. Robertson, M.A.

This topic put in a slightly different way is the question that constantly arises in one form or another—"what is the real value of mathematics to our students and what are we doing to make the high school course of most use to them—*use* in a broad sense." In any high school, be it for girls or for boys, our direct and clear course in teaching mathematics is so to present the subject that it shall stimulate the desire for knowledge for itself that is in the heart of every child, promote self activity and independence of thought, and that it shall enlist in this work the interest and enthusiasm of the child. For, instead of the old idea that interest in a subject takes the backbone out of it, we now feel that knowledge must be infused with interest if it is to reach the heart and work its influence on character.

So instead of taking in geometry, for example, the theorem all worked out in the book as the end and aim of the work and permitting our pupils to memorize the proof, we make the original the important thing from the very first page of the geometry to the last. And who, watching a class as it works out the original in perhaps half a dozen different ways and seeing the light on the faces of the students, can doubt that here assuredly is the consciousness of power following success—the intensified interest of pursuit and the rebound of intellectual pleasure over the problem solved? Here, in the immediate work itself, is all the joy of the explorer, the discoverer, and the inventor. If there is no one royal road to learning perhaps it is because all roads are royal. What though the truth be as old as the hills it is ever new to the child! This is the heart and essence of our work in algebra or geometry and it is the same in either the boys' or the girls' school.

But now, since we are convinced that no work can be made too interesting and, what is equally true, that interest in any subject is awakened and constantly reinforced by an appeal not to

books, but to life, wherever he can, the true teacher will link the subject in hand with the activities of thought-content of real life. Only let him be sure that this extraneous material be adapted to appeal to the child, and be within the range of his comprehension. A forced connection between mathematics and other subjects sheds darkness instead of light often on the work in mathematics. For after all mathematics is the subject being taught and no other. Just *here* in the application then, of mathematical truths to the outside world will come the differentiation between the course in a girls' high school as compared with that in a boys' high school. The boy, by the time he has reached high school is a bit of a mechanic. He knows lots about carpentry, engineering, wireless and constructions of various forms. Problems relating to these subjects he is hungry for. They all add dignity to the book work in mathematics for now, behold! the boy can use his geometry or his algebra in connection with his play or his outside interest. But many of these same problems are to a girl "words, words, words." The terms used, even the names of the parts of the machine, are totally unfamiliar and surely the work in mathematics is not helped but hindered. However if that same girl plays tennis, as she probably does, let her lay out a tennis court and there is at once as much enthusiasm as any boy could evince. Common sense will dictate to any teacher worthy of the name, the outside problems that are "real" to a class, and a saving sense of humor, if nothing else will prevent him from being a slave to a text book, and from offering to a class of boys the same applications offered to a class of girls. Herein lies the difference in the courses. The heart of the work is the same.

HUNTER COLLEGE HIGH SCHOOL,
NEW YORK CITY.

HOW SHOULD SECONDARY MATHEMATICS FOR GIRLS DIFFER FROM THAT FOR BOYS?

By Mary Adelle Evans.

This question implies that girls should be taught a mathematics different from that given to boys. It is fair to assume that the discrimination is made not on account of sex but on account of dissimilar vocations.

What is secondary mathematics for boys? Its widest scope includes business arithmetic; algebra through quadratics, the progressions, permutations and combinations and the other topics of advanced algebra; plane and solid geometry, trigonometry, an introduction to the calculus, and mechanics. This is a formidable list both for the average boy and for the average girl.

But a certain standardization of secondary mathematics has been brought about by the entrance requirements of the various colleges and technical schools, and by the College Entrance Board. All will agree that secondary mathematics in its limited scope includes algebra, through quadratics and the progressions, and plane geometry.

We will admit at once that for boys and girls preparing for college, technical schools or normal schools, the only difference in secondary mathematics would be occasioned by special requirements of individual higher institutions.

Our first-class high schools today offer college preparatory courses, general courses, business courses, courses preparing for normal schools, and, in some instances, vocational courses. It is already evident that with the college and normal preparatory courses this paper has nothing to do. With regard to the business course, the case is the same. Students taking a regular four year business course should not be differentiated with regard to sex in any subject. The course is not designed for developing girls as girls and boys as boys, but it aims to turn out expert specialists—stenographers, typewriters bookkeepers, etc.

Here girls are competing on the same ground as boys: their equipment must be equal.

Then, where the difference in secondary mathematics for girls and boys *may* occur, is in the general course which is not to be followed by further schooling, and in the vocational course. And this difference should be determined largely by the individual necessities and capabilities of each student.

This is only possible where there is elasticity in the curriculum and where, consequently, the course may be adjusted to the pupil, rather than an attempt made to mould the pupil to the course. In a high school where promotion is made by subject and consequently where "points" or "counts" total up for graduation, provision can be made for the incompetent in mathematics as well as for those specially adapted for its study.

In lines of education where we are not bound by restrictions and requirements of higher institutions, we may differentiate secondary mathematics to the probable advantage of the student.

We are trying to give to the boys and girls of today a broad education, that they may have some idea of life and the world *as it is* before they go out into it. We are trying to make the transition from the classroom to practical life as easy and natural as possible. It is helpful to get the viewpoint of parents on this phase of the educational question. We teachers are tempted to overvalue the importance of our own particular line of work. We are so engrossed in our own subject, it is so large a part of our waking day, that we need constantly to be on guard not to lose the perspective of daily life as it really is, outside of the ideal life of the classroom.

One father, in discussing with me secondary mathematics for boys, said: "In the first year give them practical arithmetic and *hit the high places!* Give them essential facts—not details about the important business questions. This will give to the largest number valuable information which will be immediately available if they must leave school at the end of the year to go to work."

"But where would you teach detailed business arithmetic?"

"In the second year, and more intensive work along the same line each year thereafter," he replied.

"How about algebra and geometry?"

"Make them take it."

"Regardless of course?"

"Yes: there is no business into which a high school boy may enter that will not be the better attended to by one who has had the mental discipline and training in logic and discrimination provided by the study of algebra and geometry."

"And what do you advise for girls?" I asked. His attitude was not so well defined. He advocated training in business arithmetic: for the rest—"Let those girls who wish it, study the higher mathematics."

This leads us to the two values of mathematics—the practical and the cultural. Also, there arises the question of elective courses in mathematics for girls.

Formerly no line of work requiring anything beyond very elementary mathematics was open to women. But now that women compete with men on their own ground, none will deny to the woman engineer, architect, physicist or astronomer as thorough a mathematical equipment as that demanded of the man. Consequently, this leads to the question of training in secondary mathematics for such a woman. The answer is easily found. The high school should provide for the girl the same mathematics as that given to the boy who elects such a course.

Objection will be made that a demand for such work will come rarely in a high school for girls and because of this the school should not be under obligations to provide such a course. But the present indications are that women will need more mathematics as time goes on and as larger opportunities open up to them. Far-seeing parents and ambitious pupils have a right to demand that the public school shall meet this need.

No one should understand me as advocating a *required* course for girls in such rigorous mathematical training, but I wish to go on record as advocating that an adequate course be *elective*.

In our modern "point" or "count" system a good student may find time to specialize and groups of girls frequently ask for solid geometry and some even ask for trigonometry. Provision should be made for classes in these subjects provided a minimum of, say, seven students are registered for a course. This was a regular provision in a small co-educational high school in Pennsylvania, and it is worthy of comment, that *of*

those electing the advanced mathematics from *twenty* to *thirty per cent.* were girls, who were actuated by love of the subject and ambition to teach it.

Now, if we have inferred that mathematics taught to girls should be different from that taught to boys, how are we to bring them to the point where taste is developed for solid geometry and trigonometry?

The enthusiastic teacher, the diplomatic teacher, the skillful teacher, can whet curiosity in any class, and each class may yield one or more girls eager to try a more advanced course in mathematics. With an elastic system of studies such as I mentioned earlier it would be possible to try out these girls, and in this way find groups of girls adapted to the higher work. Perhaps the teaching of the history of mathematics (girls generally like history even if they do not like mathematics) would arouse in our pupils more of an interest in the subject. We might succeed where now we fail if we could excite an interest in the development of an ancient, honorable, and perfect science and show that, if to be educated means "to know everything about something, and something about everything," neglect of the study of geometry and algebra would make one fall short of this standard.

Now, our American life is first of all practical; and then, if there is time, for the individual it may become cultural. But, despite our private opinions of what life ought to be, as educators we must prepare our pupils for life as it is. And that means that practical business arithmetic should be taught in the high school to every student for *economic protection* and to reach the largest number who can appreciate it.

Every woman should know how to keep a personal account, household accounts, how to make and receipt a bill, how to run a bank account, the meaning and use of a promissory note, the meaning of a mortgage, the advantages of savings banks, building and loan associations, various kinds of insurance, such as health, accident, life, fire burglary, etc. She should understand how to approximate foreign money equivalents, and how to calculate interest so that she may be able to approximate taxes and penalties, incomes, and interests falling due. She should know enough about stocks and bonds to read the newspaper quotations intelligently. Work in fractions of denominators lying between

2 and 16, and in three place decimals would complete an equipment for self-protection. I find that even the slowest, least interested pupil will rouse to a consideration of arithmetical questions which can be shown to be directly related to her own life and her own economic protection.

This outline of subjects would also be the minimum requirement for boys; but the difference in this branch of secondary mathematics for girls lies in the fact that this is a necessary and *sufficient* amount of arithmetic, while for boys the work should be extended as their lives will bring them more directly into touch with business. The average girl will never have use for more arithmetical drill than I have mentioned. But this much she should have in order to be able to protect her own interests, administer her own affairs, and later, probably, administer those of a home.

Just when this course in arithmetic should be given seems open for debate. If given in the first year, it reaches the greatest number of pupils but they are usually too young to appreciate the significance and value of the training. To get the best results as far as experience and ability to appreciate are concerned, would mean to give this course to the relatively few students of the fourth year. Perhaps the wisest time is the second year when the number of students is still large and where the added year of school life and perhaps a little more worldly experience will enable them to appreciate more nearly the subject matter presented. It would be advisable to have, if possible, a short review of the most important features of this course in arithmetic in the last year of the high school course.

Algebra is usually taught in the first year of high school work, and rightly so, if we are to give our girl an opportunity of finding out whether she wishes to specialize in mathematics. Here, too, the teacher can usually tell whether the student is adapted to higher work or whether, being incapable, it would be wiser for her to devote her time to something in which she can accomplish results worth while.

In its problems and applications, algebra for girls should as far as possible touch upon subject matter of special interest to them. Algebra for boys would differ only in its illustrations and applications. It is objected by some that where girls are

given only one year of algebra and then take up a business or vocational course, they get so little that they can use, that they speedily forget all about it. The defense offered is that the student must necessarily profit by a year spent in orderly and thoughtful effort, and that the attempt to generalize in accordance with certain laws is of value.

After all, the best that we can hope for from any mathematics taught to those who do not expect to specialize in it, is that the student has learned to think clearly and logically, to discriminate, to weigh, to judge, and has formed an orderly habit of mind.

Statistical graphs may be taught in connection with algebra or with arithmetic. In either case a certain amount of geometric construction would be involved. The point is that these graphs *should be taught.* Newspapers, magazines, reports, etc., are full of this kind of illustration. In this the difference between the work given to boys and to girls would be merely in the subject matter to be graphed. Methods of work would be the same. Choice of material would determine interest in the subject.

And what of vocational mathmatics? Evidently this should be closely correlated with the technical work—cooking, dressmaking, design, etc. The handling of formulas and percentages needs to be taught. The use of the algebraic equation would enter here. Solutions of simultaneous equations of the first degree in two unknowns and with decimal coefficients would occur in the study of dietetics. Certain geometric concepts enter into draughting patterns and making working plans. Here, too, those taking the home economics course may be taught the reading of meters for water, gas, electricity, and the means for determining the amount of radiating surface necessary to heat a room of given dimensions.

The arithmetic in this course would include problems based upon household equipment and administration. In this course, as well as in connection with the algebra taught to girls who are electing a commercial course, simple constructive geometry gives training to the hand and eye, teaches precision and accuracy.

In conclusion let me emphasize the following points:

1. There should be available for girls, *as an elective,* a strong course in mathematics.

2. Every girl should have an appreciative knowledge of the

essentials of business arithmetic in order to administer her own affairs.

3. A knowledge of statistical graphs is necessary to the intelligent reading of newspapers, magazines, etc.

4. Vocational mathematics should include the elements of algebra, geometry and arithmetic.

5. The difference in secondary mathematics for girls from that for boys lies in its applications and is determined by their capabilities and opportunities and not by their sex.

William Penn High School for Girls,
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WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE COLLEGE CURRICULUM?

By F. J. Holder.

(*Continued from page 112 Vol. VI.*)

cises, measurement, and experiment, nor does it preclude such practical devices as models, apparatus, photographs, cross-ruled paper, and the like. It is a practical device which serves to make easier the comprehension of mathematical truths. A reasonable number of applied problems creates an interest in the mind of the pupil that reacts strongly in increasing his understanding and appreciation of the logical side of the subject. They supply the student with a motive, and help create an interest in his work. They enable him to approach the abstract through the concrete, and assist in fixing general mathematical principles by furnishing a concrete illustration or application for many of the truths of algebra and geometry. On the other hand, to attempt to make our algebra and geometry relate too closely to the material problems of life is to make of them but chapters in a higher arithmetic, and is to lose in their study most of the value that is peculiar to them.

The most practical knowledge of any subject that one can have is a knowledge of its theory, its principle, and its methods. Such a knowledge may be called a potential knowledge. It is a stored-up aggregation of habits of thought which may be drawn upon long after an algebraic formula or a geometric theorem is forgotten. How often does an engineer have occasion to use his calculus, or how many of them could apply it if they would? And yet their mathematical training is of inestimable practical value to them in their work. Through their study of mathematics they have acquired an ability to grasp the essential elements of a problem, a capacity for accurate analysis, a power of visualization, a training of the geometric imagination, that makes possible the wonderful structures which it is their business to create.

In the growing demand of the present day for a practical education, we are inclined to strive after the obvious utilities, and to overlook the more comprehensive utilities which contribute toward an ideal and intellectual practicality, and, at the same time, are of direct value in even our bread-winning activities. Professor David Eugene Smith, of Columbia University, in "The Teaching of Geometry" presents, in forceful and persuasive language, a defence of the attitude of the conservatives toward the modifications urged by a multitude of teachers in the subject matter and method of geometry. In this connection he advocates the retention in textbooks of proofs long known to be false as well as useless. This book should prove interesting to teachers in sympathy with the modern movement and should furnish arguments to conservative teachers with which to strengthen their position.

I believe we should maintain a respectable standard of efficiency in whatever subjects we propose to include in the college curriculum. It seems to me that there is a growing tendency to lessen the requirements in the regular daily and term work and to give the student the impression that his final grade depends too largely upon his attendance and deportment without due consideration of his scholarly achievements. We claim to give our courses in accordance with the ability of the average of the class while, in reality, they measure up only to the standard of the poorer students and, in many cases, fall far short of encouraging and stimulating the good students to their best efforts. We are so taken up with the care and attention of the ill prepared, to prevent their falling by the wayside, that we often neglect, and fail to develop, our very best material. But, you say, "What about our *required* courses in which we have to deal, not merely with the mathematically inclined, but with those unfortunate specimens of humanity whom the Lord never intended for mathematicians?" That is just the point I want to make. I grant that they should be given all due consideration, help, sympathy, and encouragement, for, whether we feel free to openly acknowledge it or not, we are more or less impressed with the fact that the dean and the president have gone out into the highways and hedges and brought in these specimens and, while it is their duty to get all they can, they have employed us

to keep all they get. Undoubtedly, this idea of bigness, of name and fame, is making its impression upon our American colleges and universities, and the ambition to be great and popular is not monopolized by the dean and president, but it is entertained by the members of the faculty as well. I believe we, as a faculty, should feel a responsibility for the entire student body and should endeavor to develop our best, as well as our poorest, material, and if it turns out that the dean and the president, in gathering up their raw recruits, have picked one of their fruits too green and, after giving it a fair and impartial chance to ripen with all the cultivation we have to offer, we are convinced that it is a dwarf, not only in mathematics but in every other subject in the curriculum, these same officials will be the first men to stand by us in sending this individual back to the orchard for further growth and maturity, and this deed will be a blessing to him as well as to those who remain.

I believe that a college degree should mean more than a mere four years' attendance on the campus, an athletic letter, a Greek letter fraternity pin, and the ability to drink a glass of Budweiser without removing the half-inch cigarette stub from the other corner of the mouth. It should mean that the one who bears the college seal and stamp has learned to apply himself to the task to be done, to meet and overcome difficulties, to recognize the rights of others and to sympathize with the less fortunate, and that he is possessed with the determination to use his powers of concentration, logical reasoning, and sound judgment, for the betterment of humanity and the progress of civilization.

I know of no department in the college better adapted and more fully equipped to set the pace in this direction than that of mathematics, and to this end we should have as our motto: "Give the student something worth while, expect a reasonable response, and then see that we get it."

University of Pittsburgh,
Pittsburgh, Pa.

THE MATHEMATICAL TRAINING OF GIRLS IN GERMANY.

By Carrie Butz.

The present system of secondary schools for girls in Prussia is comparatively new. It may be said to date from August, 1908, with slight modifications in February, 1912, as to terminology.

Before 1908, provision had been made for the establishment of higher girls' schools and normal schools, but little for the further training of girls after completing the higher school.

It is true that the Prussian government through the famous regulations, May 31, 1894, über das Mädchenschulwesen, die Lehrerinnenbildung und die Lehrerinnenprüfung, had prescribed a nine years' course and special elective courses for the education of girls after the completion of this course, but these regulations were not adequate to meet the wants and needs for the proper education of girls. Almost nothing was done for the future housewife and mother, or for the girl who desired to go to the university.

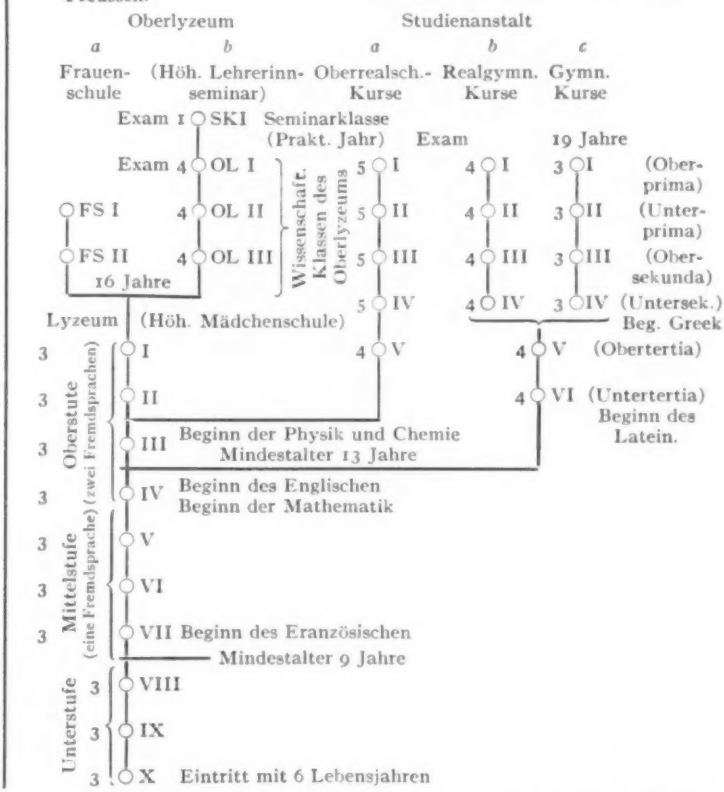
Before 1908, teaching conditions were not regulated and it was difficult to secure the best teachers for girls' schools. The Prussian government, however, in 1908, decreed that the higher girls' school be placed officially under the control of the Provincial School Board and that the Direktors and Oberlehrer who taught at these schools should take equal rank (salaries and pensions) with the Direktors and Oberlehrer of boys' schools. It also decreed that the course be changed from nine to ten years and that one half of the periods in the academic subjects of the upper and middle grades be taught by university trained teachers. The girls' schools were thus put on the same basis as the boys' schools.

The higher girls' school or Lyzeum, as termed by the provision of February, 1912, has ten classes designated by X (the lowest) to I (the highest) as shown in the accompanying sketch

or table. The minimum age at which a girl may enter the lowest class is six years. At nine she begins French, at twelve, the study of English and of mathematics as distinguished from Rechnen, to which time is given during the first six years.

The work for the different classes of the Lyzeum is as follows:

1. Die Organisation höheren Bildungsanstalten für die weibliche Jugend in Preussen.



Classes X–VIII (av. age 6 to 8).—Mental reckoning with numbers from 1 to 1,000. Written exercises in the four fundamental operations and easy examples in the rule-of-three.

Class VII (av. age 9).—The four fundamental operations. German weights, measures and money. Practice in writing

decimals. Easy examples in decimals and in the rule-of-three. Reduction of easy fractions from higher to lower terms.

Class VI (av. age 10).—Decimals continued. Divisibility of numbers. Greatest common divisor and least common multiple. Common fractions and familiarity with the more common solids.

Class V (av. age 11).—Decimal fractions. Simple and compound proportion. Percentage and interest. Introduction of algebraic symbols for arithmetical numbers. Simple problems in surfaces and solids.

(In the seventh school year mathematics begins.)

Class IV (av. age 12).—Addition, subtraction, multiplication and division of algebraic quantities. Simple equations of the first degree with one unknown quantity. Propadeutic work in plane geometry. Most important properties of triangles. Learning of definitions and forms.

Class III (av. age 13).—Division and fractions (algebraic numbers). Factoring. Equations of the first degree. Constructions. Triangles, and the parallelogram and trapezoid.

Class II (av. age 14).—Equations of the first degree with two unknown quantities. Graphic representation of the function of the first degree. Proportion. The circle. The measurements of rectilinear figures and the theorem of Pythagoras.

Class I (av. age 15).—Square root of numbers. Simple equations of the second degree with one unknown quantity. Graphic representation of these functions. Regular polygons. The circumference and area of the circle. Calculation of volume and surface of simple solids.

After completing the ten years' course in the Lyzeum, the girl who wishes may enter the Frauenschule for two years. Her work here is such as to fit her to take care of a home and includes no mathematics. The girl who wishes to prepare to teach, may enter the academic classes of the Oberlyzeum for three years. She continues her work in mathematics for four hours each week for three years and finishes with a Seminar Klasse or Practical Year.

O L III (age 16–17).—Theory of powers, roots and logarithms.

Equations of the second degree. Similarity. Proportion. Constructions with algebraic analysis.

O L II (age 17–18).—Arithmetical and geometrical series. Interest and annuities. Equations of the second degree with two unknown quantities. Introduction to the study of harmonic points, pencils and transversals. Trigonometry.

O L I (age 18–19).—Review. Binomial theorem for positive integral exponents. Stereometry.

Practical year (age 19–20).—Methods of teaching and introduction to the literature of the subject. The fundamentals of plane analytical geometry.

Should the girl desire a university education, she could enter one of the three Studienanstalten or University Preparatory Schools.

The Oberrealschule affords more opportunity for mathematics and science and may be entered after eight years in the Lyzeum. Physics and chemistry are begun in class III of the Lyzeum and then five years would be spent doing the work of the Oberrealschule. At nineteen years of age the girl would be ready to enter the University. (The Arabic numerals to the left of any course in the sketch indicate the number of hours per week devoted to mathematics.)

Should the girl desire less science and more of the languages, she would leave the Lyzeum at the end of the first seven years and enter the Realgymnasium, if she wanted Latin but no Greek, or the Gymnasium, if she desired to study both Latin and Greek.

Mathematics of the Oberrealschule.

Class V (age 14–15).—Equations of the first degree with several unknown quantities. Proportion. Square root. Easy equations of the second degree with one unknown quantity. The circle. Measurement of rectilinear figures.

Class IV (age 15–16).—Theory of powers, roots and logarithms. Equations of second degree with one and two unknown quantities. Similarity. Regular polygons. The circumference and area of the circle. Trigonometry of triangles and polygons.

Class III (age 16–17).—Equations of the second degree with two and more unknown quantities. Arithmetical and geo-

metrical series. Interest and annuities. Trigonometry. Stereometry with special reference to the elements of projection. Harmonic points, pencils, transversals. Axes of symmetry.

Classes II and I (age 17–19).—Equations of the third degree. Binomial theorem for any exponent. The most impotant series. Maxima and minima. Combinations and their application. A complete review of all works. Constructions with algebraic analysis. Conic sections, a synthetic and analytic treatment. Spherical trigonometry, sufficient to be able to understand mathematical geography.

Mathematics of the Realgymnasium.

Class VI (age 13–14).—Division and fractions of algebraic numbers. Factoring. Equations of the first degree. The triangle, parallelogram and trapezoid.

Class V (age 14–15).—Equations of the first degree with two or more unknown quantities. Graphic representation of functions of the first degree. Simple theorems of proportion. Square root. Easy equations of the second degree with one unknown quantity. The circle. Measurement of rectilinear figures.

Classes IV–I (age 15–19).—Same as that of the Oberrealschule, only four hours per week devoted to the study of mathematics instead of five, and conic sections are treated only analytically.

Mathematics of the Gymnasium.

Classes VI and V (age 13–15).—Same as the Realygymnasium.

Class IV (age 15–16).—Theory of powers, roots logarithms. Similarity. Regular polygons. Measurement of the circumference and area of the circle.

Class III (age 16–17).—Equations of the second degree with two unknown quantities. Transversals, harmonic points and pencils. Trigonometry of the triangle.

Classes II and I (age 17–19).—Arithmetical and geometrical series. Interest and annuities. Binomial theorem for positive integral exponents. Complete review of all previous work. Stereometry. Fundamentals of the conic sections.

At the completion of the university preparatory courses, the girl must pass an examination. Five hours are given to the written examination, which may be divided into two parts of two and one half hours each. The written examination is followed a few weeks later by an oral examination. The best pupils may be excused from the oral examination providing their classwork and written examination has been meritorious. The same regulations as to examinations apply to those who have completed the three academic years of the Oberlyzeum.

Examination Questions Given at the Realgymnasium at Cassel.

1. Determine the roots: $x^4+4x^3-13x^2-4x+12=0$; discuss methods used.

2. What is the coefficient of $x^{n-r}y^r$ in the expansion $(x-\frac{1}{2}y)^n$, when n is the third term of a geometrical series of six terms, the sum of whose even terms is 147, and the sum of whose odd terms is $73\frac{1}{2}$? r is the number of terms of an arithmetical series, the first term of which is 5, the common difference 3, and the sum is 98.

3. Determine the distance between Vienna (lat. 48° 12′ 36″, longitude 34° 2′ 36″) and Paris (lat. 48° 50′ 11″, long. 20°). Determine the remaining parts of the spherical triangle.

4. Determine the equation and the area of a circle which passes through the point 4, 3, and is externally tangent to the circle, $(x-2)^2+y^2=1$ and also tangent to the axis of ordinates.

(This is not any specially selected list of questions but the first set of a number mentioned by Dr. Schröder.)

The courses for girls in Saxony, Bavaria, Hesse, Baden and ohter parts of Germany, follow, in the main, the work as prescribed for the girls' schools of Prussia.

They all give ten years to the Lyzeum and all provide for the further training of girls, either in Frauenschulen or university preparatory schools, or both. In most countries, girls, at the age of nineteen, are ready to enter the university. In Baden the Oberrealschule at Mannheim takes the girl after six years in the Lyzeum and in six years more prepares her for the University. She is then eighteen years old. This is done by devoting

from five to seven hours a week to mathematics during the last six years. During the last two school years the work is as follows:

Unterprima (age 16–17).—Combinations. Binomial theorem. Complex numbers. Equations of higher degrees, especially cubics. Graphs of these functions. Equations of the circle, parabola and ellipse. Different equations of the straight line. Differential quotients of $y = ax^n$, $\sin x$, $\cos x$, $\tan x$, $\log x$, $y = w + v$; $v \cdot w$, $y = \text{arc} \sin x$. Stereometry. Spherical trigonometry. Orthogonal projection.

Oberprima (age 17–18).—Maxima and minima. Infinite series. Integration and summation of areas. Harmonic points and pencils. Conic sections as geometrical loci. Plane analytics. Sections of solids.

For any more extended information on this subject, see Die Neuzeitliche Entwicklung des Mathematischen Unterrichts an den Höheren Mädchenschulen Deutschlands, von Dr. J. Schröder, Direktor des Staatlichen Lyzeums am Lerchenfeld in Hamburg. Published by B. G. Teubner, Leipsig and Berlin, 1913.

The question arises whether we, in our country, can learn anything from the Germans concerning the mathematical training of girls. Their old curriculum, before 1908, laid too much stress on training the feeling and not enough on training the understandig.

Their present courses for girls would indicate that they believe in the girl's ability to learn mathematics, as well as in the necessity of such training, since girls there are beginning to fill the same places as boys.

We hear a great deal here in America about making the mathematics easy. Does the girl in our country need to be less well prepared along these lines? Do the Germans teach too much mathematics? Should we teach more? These are questions I shall leave to the reader to investigate.

TEACHERS COLLEGE,
NEW YORK CITY.

NEW BOOKS.

Descriptive Geometry. Part I, Lines and Planes. By JOHN C. TRACY. **Part II, Solids.** By HERBERT B. NORTH and JOHN C. TRACY. New York: John Wiley and Sons. Pp. 126. $2.00.

The author considers that there are only four problems in descriptive geometry that are fundamentally different; all others depend for their solution upon one or more of these four fundamental problems. This fact, becoming apparent during a long experience in teaching, led to the preparation of Part I of this book, which, in reprint form, has been used as a text-book for several years. The addition of Part II completes a book in which the main object is to teach the student to resolve a new problem into its component parts or steps, and to recognize in each step a previous problem with which he is already familiar. This is a method of attack that will help to simplify not only problems in descriptive geometry, but problems in other engineering subjects as well.

Elementary Experimental Dynamics. By C. E. ASHFORD. Cambridge: The University Press. Pp. 246. $1.25.

This and a companion volume on "Elementary Experimental Statics," soon to appear, form a course of mechanics for boys. The author instead of starting with Newton's Laws as axioms and developing the subject deductively, starts with simple quantitative experiments and develops the fundamental principles of mechanics from the results. More explicitly mathematical processes are introduced later and in the end the student should have a very intelligent comprehension of the elements of the subject. The text presents an attractive appearance and is written in a lucid style.

The Twisted Cubic. By P. W. WOOD. Cambridge: The University Press. Pp. 78. 75 cents.

This little volume fills a need due to the fact that the twisted cubic has been so little treated in English from a fundamental and connected point of view. In the first part the projective properties of twisted cubics are treated and in the second part the relations between the asymptotes, the diameters and other elements associated with the cubical hyperbola are dealt with from a metrical standpoint. The treatment presumes some knowledge of both projective and analytical methods and furnishes a good introduction to the wide field of twisted curves.

The Theory of Numbers. By ROBERT D. CARMICHAEL. New York: John Wiley and Sons. Pp. 94. $1.00.

This is number 13 of the Mathematical Monographs issued by the publishers. In the first five chapters a treatment of the elementary proper-

ties of integers is given. These chapters also cover Congruences, Fermat's and Wilson's theorems and primitive roots. In the last chapter the author gives a brief treatment of the theory of quadratic residues, Galoi's imaginaries, analytic theory of numbers, Diophantine equations, Pythagorean triangles and the equation $x^n + y^n = z^n$.

Elementary Theory of Equations. By LEONARD E. DICKSON. New York: John Wiley and Sons. Pp. 184. $1.75.

This book, as the title implies, gives an elementary treatment of the field usually covered by an introductory course. It starts out with a good treatment of graphs and proceeds to complex numbers by means of vectors. In the solution of numerical equations Newton's method is used in preference to Horner's. The work seems to be carefully done in spite of the following sentence which appears in the first paragraph: "One may be sure that a given cubic equation has only the one real root seen in the graph if the bend points lie on the opposite sides of the x-axis."

Arithmetic. Book I, Fundamental Processes; Book II, Practical Applications. By JOHN H. WALSH and HENRY SUZZALLO. Book I, 35 cents. Book II, 65 cents. Boston: D. C. Heath & Co.

Among the urgent demands now made regarding instruction in arithmetic are that (1) fundamental processes shall be emphasized in the lower grades in order that efficiency may result; (2) that the social and economic applications of arithmetic shall be taught in the upper grades so that grammar school children will have an insight into the typical practices of modern life. These books seem to meet both requirements to an exceptional degree.

The series is so arranged that a pupil may acquire an easy and accurate command of the processes by the end of the sixth year. The seventh and eighth school years are thus left free for the study of practical applications. Few if any books heretofore offered to schools contain so varied and extended a series of applications of arithmetic suitable to the conditions of modern life such as the rank and file of pupils are likely to meet.

Francis W. Parker School Year Book. Volume III, June, 1914. 188 pages. 50 illustrations. Francis W. Parker School, Chicago.

This volume, prepared by the faculty of the Francis W. Parker School, Chicago, deals with "Expression as a Means of Developing Motive," or the place of expression in the process of education. It is a distinctive contribution to literature on social education, and portrays vividly certain fundamental phases of education as they have been worked out in this school. Those who have read Volumes I and II of this *Year Book* will welcome the present volume.

NOTES AND NEWS.

THE Chicago Branch of Keuffel & Esser Co., of New York, have now completed its removal to its new quarters, in the seven-story K. & E. Building, 516–518–520 South Dearborn Street, Chicago. The company recently purchased this building and occupies the greater part of it. It is situated near the "loop" and half a block from an "L" station, being midway between Van Buren and Harrison Streets, on the west side of South Dearborn Street; within half a block of the Fisher Building, the Monadnock Block, Old Colony Building, Transportation Building, in all of which are the offices of many K. & E. customers. A visit to the splendidly equipped new store (including the complete blueprint department, etc.), so much more commodious and convenient than the old location, 68 West Madison Street, will repay anyone interested in the Keuffel & Esser Co. line.

George W. Richardson, of Chicago, puts out a slide rule which has many excellent features. Among them are a key hole whereby a red letter known as a key on the slide is inserted in the keyhole, making the answer direct reading for that particular problem, and has scales for adding and subtracting. This rule should be seen to be fully appreciated. He also makes a folding pocket rule which can be used as a hook-rule, caliper-guage, protractor, triangle or tri-square.

NEW MEMBERS.

UNANGST, HARRY G.; 116 Hawkins Ave., Braddock, Pa.
CARRUTH, WILLIAM MASSEY, Hamilton College, Clinton, N. Y.
COBANE, MISS M. E., Skeneateles, N. Y.
MORRIS, RICHARD, Rutgers College, New Brunswick, N. J.
BLAIR, MISS VEVIA, 17 East 60th St., New York City.
BUTLER, MISS HILDA WALL, 4 East Ave., Albion, N. Y.
CLAPP, MORRIS B., Cincinnatus, N. Y.
BURRITT, EDNA C., 22 West 34th St., Bayonne, N. J.
CRANBELT, N. B., University of Rochester, Rochester, N. Y.
HAMMOND, MISS IDA, State Normal School, Mansfield, Pa.
HARDY, MISS ELIZA G., 45 E. Washington Lane, Germantown, Pa.
HARRY, STEPHEN CLOUD, 1721 McCullon St., Baltimore, Md.
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